Subsidising Inclusive Insurance to Reduce Poverty

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Abstract

In this article, we consider a compound Poisson-type model for households' capital. Using risk theory techniques, we determine the probability of a household falling under the poverty line. Microinsurance is then introduced to analyse its impact as an insurance solution for the lower income class. Our results validate those previously obtained with this type of model, showing that microinsurance alone is not sufficient to reduce the probability of falling into the area of poverty for specific groups of people, since premium payments constrain households' capital growth. This indicates the need for additional aid particularly from the government. As such, we propose several premium subsidy strategies and discuss the role of government in subsidising microinsurance to help reduce poverty.

Keywords— microinsurance; poverty traps; trapping probability; cost of social protection; government subsidies.

1 Introduction

Inclusive insurance (or microinsurance) relates to the provision of insurance services to low-income populations with limited access to mainstream insurance or alternative effective risk management strategies. Many individuals excluded from basic financial services and those microinsurance aims to proctect, live below the minimum level of income required to meet their basic needs. Currently fixed at $1.90 USD per day, 9.2% of the population were estimated to live below the international extreme poverty line in 2017 (**?**). Increases in the number of new poor and those returning to poverty as a result of the COVID-19 pandemic are expected to reverse the historically declining poverty trend (**?**).

Fundamental features of the microinsurance environment such as the nature of low income risks, limited financial literacy and experience, product accessibility and data availability, create barriers to penetration, particularly in relation to the affordability of products. For the proportion of the population living just above the poverty line, premium payments heighten the risk of poverty trapping and induce a balance between profit and loss as a result of insurance coverage, dependent on the entity's level of capital. Here, poverty trapping refers to the inability of the poor to escape poverty without external help (Kovacevic and Pflug, 2011).

Highlighting vulnerability reduction and investment incentive effects of insurance, Janzen et al. (2020) observe a marked reduction in long-term poverty and the social protection costs required to close the poverty gap following introduction of an asset insurance market. Calibrating their model to risk-prone regions in Africa, their study suggests that those in the neighbourhood of the poverty line do not optimally purchase insurance (without subsidies), suppressing their consumption and mitigating the probability of trapping. Kovacevic and Pflug (2011) propose negative consequences of insurance uptake for members of low-income populations closest to the poverty line, applying ruin-theoretic approaches to calculation of the trapping probability. **?** support these findings in their analysis of a multi-equilibrium model with agricultural output risks on data from rural China. Voluntary insurance would enable individuals close to the poverty threshold to opt out of insurance purchase in favour of alternative risk management strategies, in order to mitigate this risk.

In line with the findings of **?** on the effectiveness of social protection mechanisms for poverty alleviation, **?** observe a greater reduction in poverty through implementation of an integrated social protection programme in comparison to pure cash transfers. Government subsidised premiums are the most common form of aid in the context of insurance. Besides reducing the impact on household capital growth, lowering consumer premium payments has the potential to increase microinsurance take-up, with wealth and product price positively and negatively influencing microinsurance demand, respectively (Eling et al., 2014).

Poverty traps are typically studied in the context of economics, with a large literature focus on why economic stagnation below the poverty line occurs in certain communities. While the poor could readily grow their way out of poverty by adopting profitable strategies such as productive asset accumulation, opportunistic exchange and implementation of cost-effective production technologies, poverty traps are underlined by poverty reinforcing behaviours induced by the state of being poor (**?**). A detailed description of the mechanics of the poverty trap state is provided by **?**. In studying the probability of falling into such a trap, "trapping" describes the event in which a household falls underneath the poverty line and into the area of poverty.

In this paper, we adopt the ruin-theoretic approach to calculating the trapping probability of households in low-income populations presented by Kovacevic and Pflug (2011), adapting the piecewise deterministic Markov process such that households are subject to large shocks of random size. In line with the poverty trap ideology, we assume that the area of poverty to be an absorbing state and so consider only the state of events above the poverty threshold. Obtaining explicit solutions for the trapping probability, we compare the influence of three structures of microinsurance on the ability of households to stay above the poverty line. Specifically, we consider a (i) proportional, (ii) subsidised proportional and (iii) subsidised proportional with barrier microinsurance scheme. Aligning with the essential place for governmental support in the provision of social protection which encompasses risk mitigation, we assess for the first time in this context, to the best of our knowledge, the impact of a (government) subsidised insurance scheme with barrier strategy. We optimise the barrier level in the context of the trapping probability and the governmental cost of social protection, identifying the proportion of the population for which such a product would be beneficial. Here, the cost of social protection is defined to account for the provision of government subsidies, in addition to the cost of lifting a household from poverty, should they fall underneath the threshold. The benefit of subsidy schemes for poverty reduction is measured through observation of this governmental cost, in addition to the trapping probability of the households under consideration.

The remainder of the paper will be structured as follows. In Section 2, we introduce the household capital model and its associated infinitesimal generator. The (trapping) time at which a household falls into the area of poverty is defined in Section 3, and subsequently the explicit trapping probability and the expected trapping time are derived for the basic uninsured model. Links between classical ruin theoretic models and the trapping model of this paper are stated in Sections 2 and 3. Microinsurance is introduced in Section 4, where we assume a proportion of household losses are covered by a microinsurance policy. The capital model is redefined and the trapping probability is derived. Sections 5 and 6 consider the case where households are proportionally insured through a government subsidised microinsurance scheme, with the impact of a subsidy barrier discussed in Section 6. Optimisation of the subsidy and barrier levels is presented in Sections 5 and 6, alongside the associated governmental cost of social protection. Concluding remarks are provided in Section 7.

2 The Capital Model

The fundamental dynamics of the model follow those of Kovacevic and Pflug (2011), where the growth in accumulated capital (X_t) of an individual household is given by

$$\frac{dX_t}{dt} = r \cdot [X_t - x^*]^+ , \tag{2.1}$$

where $[x]^+ = \max(x, 0)$. The capital growth rate r incorporates household rates of consumption, income generation and investment or savings, while $x^* > 0$ represents the threshold below which a household lives in poverty. Reflecting the ability of a household to produce, accumulated capital (X_t) is composed of land, property, physical and human capital, with health a form of capital in extreme cases where sufficient health services and food accessibility are not guaranteed (Dasgupta, 1997). The notion of a household in this model setting may be extended for consideration of poverty trapping within economic units such as community groups, villages and tribes, in addition to the traditional household structure.

The dynamical process in (2.1) is constructed such that consumption is assumed to be an increasing function of wealth (for full details of the model construction see Kovacevic and Pflug (2011)). The poverty threshold x^* represents the amount of capital required to forever attain a critical level of income, below which a household would not be able to sustain their basic needs, facing elementary problems relating to health and food security. Throughout the paper, we will refer to this threshold as the critical capital or the poverty line. Since (2.1) is positive for all levels of capital greater than the critical capital, points less than or equal to x^* are stationary (capital remains constant if the critical level is not met). In this basic model, stationary points below the critical capital are not attractors of the system if the initial capital exceeds x^*, in which case the capital process (X_t) grows exponentially with rate r.

Using capital as an indicator of financial stability over other commonly used measures such as income enables a more effective analysis of a household's wealth and well-being. Households with relatively high income, considerable debt and few assets would be highly vulnerable if a loss of income was to occur, while low-income households could live comfortably on assets acquired during more prosperous years for a long-period of time (Gartner et al., 2004).

In line with Kovacevic and Pflug (2011), we expand the dynamics of (2.1) under the assumption households are susceptible to the occurrence of large capital losses, including severe illness, the death of a household member or breadwinner and catastrophic events such as floods and earthquakes. We assume occurrence of these events follows a Poisson process with intensity λ, where the capital process follows the dynamics of (2.1) between events. On the occurrence of a loss, the household's capital at the event time reduces by a random amount Z_i. The sequence (Z_i) is independent of the Poisson process and i.i.d. with common distribution function G. In contrast to Kovacevic and Pflug (2011), we assume reduction by a given amount rather than a random proportion of the capital itself. This adaptation enables analysis of a tractable mathematical model without threatening the core objective of studying the probability that a household falls into the area of poverty.

A household reaches the area of poverty if it suffers a loss large enough that the remaining capital is attracted into the poverty trap. Since a household's capital does not grow below the critical capital x^*, households that fall into the area of poverty will never escape. Once below the critical capital, households are exposed to the risk of falling deeper into poverty, with a risk of negative capital due to the dynamics of the model. A reduction in a household's capital below zero could represent a scenario where total debt exceeds total assets, resulting in negative capital net worth. The experience of a household below the critical capital is, however, out of the scope of this paper.

We will now formally define the stochastic capital process, where the process for the inter-event household capital (2.2) is derived through solution of the first order ordinary differential equation (2.1). This model is an adaptation of the model proposed by Kovacevic and Pflug (2011).

Definition 1. *Let T_i be the i^{th} event time of a Poisson process (N_t) with parameter λ, where $T_0 = 0$. Let $Z_i \geq 0$ be a sequence of i.i.d. random variables with distribution function G, independent of the process (N_t). For $T_{i-1} \leq t < T_i$, the stochastic growth process of the accumulated capital X_t is defined as*

$$X_t = \begin{cases} \left(X_{T_{i-1}} - x^*\right) e^{r(t-T_{i-1})} + x^* & \text{if } X_{T_{i-1}} > x^*, \\ X_{T_{i-1}} & \text{otherwise.} \end{cases} \tag{2.2}$$

At the jump times $t = T_i$, the process is given by

$$X_{T_i} = \begin{cases} \left(X_{T_{i-1}} - x^*\right) e^{r(T_i-T_{i-1})} + x^* - Z_i & \text{if } X_{T_{i-1}} > x^*, \\ X_{T_{i-1}} - Z_i & \text{otherwise.} \end{cases} \tag{2.3}$$

The stochastic process $(X_t)_{t\geq 0}$ is a piecewise-determinsitic Markov process (Davis, 1984) and its infinitesimal generator is given by

$$(\mathcal{A}f)(x) = r(x - x^*)f'(x) + \lambda \int_0^\infty [f(x-z) - f(x)]\,\mathrm{d}G(z), \qquad x \geq x^*.$$

The capital model as defined in (2.2) and (2.3) is actually a well-studied topic in ruin theory since the 1940s. Here, modelling is done from the point of view of an insurance company. Consider the insurer's surplus process $(U_t)_{t\geq 0}$ given by

$$U_t = u + pt + a\int_0^t U_s\,ds - \sum_{i=1}^{N_t} Z_i, \tag{2.4}$$

where u is the insurer's initial capital, p is the constant premium rate, a is the risk-free interest rate, N_t is a Poisson process with parameter λ which counts the number of claims in the time interval $[0,t]$, and $(Z_i)_{i=1}^{\infty}$ is a sequence of i.i.d. claim sizes with distribution function G. This model is also called the insurance risk model with deterministic investment, which was first proposed by Segerdahl (1942) and subsequently studied by Harrison (1977) and Sundt and Teugels (1995). For a detailed literature review on this model prior to the turn of the century, readers can consult Paulsen (1998).

Observe that when $p = 0$, the insurance model (2.4) for positive surplus is equivalent to the capital model (2.2) and (2.3) above the poverty line $x^* = 0$. Subsequently, the capital growth rate r in our model corresponds to the risk-free investment rate a of the insurer's surplus model. More connections between these two models will be made in the next section after the first hitting time is introduced.

3 The Trapping Time

Let

$$\tau_x := \inf\left\{t \geq 0 : X_t < x^* \mid X_0 = x\right\}$$

denote the time at which a household with initial capital $x \geq x^*$ falls into the area of poverty (the trapping time), where $\psi(x) = \mathbb{P}(\tau_x < \infty)$ is the infinite-time trapping probability. To study the distribution of the trapping time, we apply the expected discounted penalty function at ruin concept commonly used in actuarial science (Gerber and Shiu, 1998), such that with a force of interest $\delta \geq 0$ and initial capital $x \geq x^*$, we consider

$$m_\delta(x) = \mathbb{E}\left[w(\mid X_{\tau_x} - x^* \mid) e^{-\delta\tau_x} \mathbb{1}_{\{\tau_x<\infty\}}\right], \tag{3.1}$$

where $\mid X_{\tau_x} - x^* \mid$ is the deficit at the trapping time and $w(x)$ is an arbitrary non-negative penalty function. For more details on the so called Gerber-Shiu risk theory, the interested reader may wish to consult Kyprianou (2013). Using standard arguments based on the infinitesimal generator, $m_\delta(x)$ can be characterised as the solution of the Integro-Differential Equation (IDE)

$$r(x-x^*)m_\delta'(x)-(\lambda+\delta)m_\delta(x)+\lambda\int_0^{x-x^*}m_\delta(x-z)dG(z)=-\lambda A(x), \qquad x\geq x^*, \tag{3.2}$$

where

$$A(x):=\int_{x-x^*}^{\infty}w(z-x)dG(z).$$

Due to the lack of memory property, we consider the case in which losses (Z_i) are exponentially distributed with parameter $\alpha>0$. Specifying the penalty function such that $w(x)=1$, $m_\delta(x)$ becomes the Laplace transform of the trapping time, also interpreted as the expected present value of a unit payment due at the trapping time. Equation (3.2) can then be written such that

$$r(x-x^*)m_\delta'(x)-(\lambda+\delta)m_\delta(x)+\lambda\int_0^{x-x^*}m_\delta(x-z)\alpha e^{-\alpha z}dz=-\lambda e^{-\alpha(x-x^*)}, \quad x\geq x^*. \tag{3.3}$$

Applying the operator $\left(\frac{d}{dx}+\alpha\right)$ to both sides of (3.3), together with a number of algebraic manipulations, yields the second order homogeneous differential equation

$$-\frac{(x-x^*)}{\alpha}m_\delta''(x)+\left[\frac{(\lambda+\delta-r)}{\alpha r}-(x-x^*)\right]m_\delta'(x)+\frac{\delta}{r}m_\delta(x)=0, \qquad x\geq x^*. \tag{3.4}$$

Letting $f(y):=m_\delta(x)$, such that y is associated with the change of variable $y:=y(x)=-\alpha(x-x^*)$, (3.4) reduces to Kummer's Confluent Hypergeometric Equation (Slater, 1960)

$$y\cdot f''(y)+(c-y)f'(y)-af(y)=0, \qquad y<0, \tag{3.5}$$

for $a=-\frac{\delta}{r}$ and $c=1-\frac{\lambda+\delta}{r}$, with regular singular point at $y=0$ and irregular singular point at $y=-\infty$ (corresponding to $x=x^*$ and $x=\infty$, respectively). A general solution of (3.5) is given by

$$m_\delta(x)=f(y)=\begin{cases}1 & x<x^*,\\ A_1M\left(-\frac{\delta}{r},1-\frac{\lambda+\delta}{r};y(x)\right)+A_2e^{y(x)}U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};-y(x)\right) & x\geq x^*,\end{cases} \tag{3.6}$$

for arbitrary constants $A_1,A_2\in\mathbb{R}$. Here,

$$M(a,c;z) = {}_1F_1(a,c;z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n} \frac{z^n}{n!}$$

is Kummer's Confluent Hypergeometric Function (Kummer, 1837) and $(a)_n = \frac{\Gamma(a+n)}{\Gamma(n)}$ denotes the Pochhammer symbol (Seaborn, 1991). In a similar manner,

$$U(a,c;z) = \begin{cases} \frac{\Gamma(1-c)}{\Gamma(1+a-c)} M(a,c;z) + \frac{\Gamma(c-1)}{\Gamma(a)} z^{1-c} M(1+a-c, 2-c; z) & c \notin \mathbb{Z}, \\ \lim_{\theta \to c} U(a,\theta;z) & c \in \mathbb{Z} \end{cases}$$

is Tricomi's Confluent Hypergeometric Function (Tricomi, 1947). This function is generally complex-valued when its argument z is negative, i.e. when $x \geq x^*$ in the case of interest. We seek a real-valued solution of $m_\delta(x)$ over the entire domain, therefore an alternative independent pair of solutions, here, $M(a,c;z)$ and $e^z U(c-a,c;-z)$, to (3.5) are chosen for $x \geq x^*$.

To determine the constants A_1 and A_2, we use the boundary conditions at x^* and at infinity. Applying equation (13.1.27) of Abramowitz and Stegun (1972), also known as Kummer's Transformation $M(a,c;z) = e^z M(c-a,c;-z)$, we write (3.6) such that

$$m_\delta(x) = e^{y(x)} \left[A_1 M \left(1 - \frac{\lambda}{r}, 1 - \frac{\lambda+\delta}{r}; -y(x) \right) + A_2 U \left(1 - \frac{\lambda}{r}, 1 - \frac{\lambda+\delta}{r}; -y(x) \right) \right] \quad (3.7)$$

for $x \geq x^*$. For $z \to \infty$, it is well-known that

$$M(a,c;z) = \frac{\Gamma(c)}{\Gamma(a)} e^z z^{a-c} \left[1 + O\left(|z|^{-1} \right) \right]$$

and

$$U(a,c;z) = z^{-a} \left[1 + O\left(|z|^{-1} \right) \right]$$

(see for example, equations (13.1.4) and (13.1.8) of Abramowitz and Stegun (1972)). Asymptotic behaviours of the first and second terms of (3.7) as $y(x) \to -\infty$ are therefore given by

$$\frac{\Gamma\left(1 - \frac{\lambda+\delta}{r}\right)}{\Gamma\left(1 - \frac{\lambda}{r}\right)} (-y(x))^{\frac{\delta}{r}} \left(1 + O\left(|-y(x)|^{-1} \right) \right) \quad (3.8)$$

and

$$e^{y(x)}\left(-y(x)\right)^{\frac{\lambda}{r}-1}\left(1+O\left(|-y(x)|^{-1}\right)\right), \tag{3.9}$$

respectively. For $x \to \infty$, (3.8) is unbounded, while (3.9) tends to zero. The boundary condition $\lim_{x\to\infty} m_\delta(x) = 0$, by definition of $m_\delta(x)$ in (3.1), thus implies that $A_1 = 0$. Letting $x = x^*$ in (3.3) and (3.6) yields

$$\frac{\lambda}{(\lambda+\delta)} = A_2 U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; 0\right).$$

Hence, $A_2 = \frac{\lambda}{(\lambda+\delta)U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};0\right)}$ and the Laplace transform of the trapping time is given by

$$m_\delta(x) = \frac{\lambda}{(\lambda+\delta)U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};0\right)} e^{y(x)} U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; -y(x)\right). \tag{3.10}$$

Remarks.

(i) Figure 1(a) shows that the Laplace transform of the trapping time approaches the trapping probability as δ tends to zero, i.e.

$$\lim_{\delta\downarrow 0} m_\delta(x) = \mathbb{P}(\tau_x < \infty) \equiv \psi(x).$$

As $\delta \to 0$, (3.10) yields

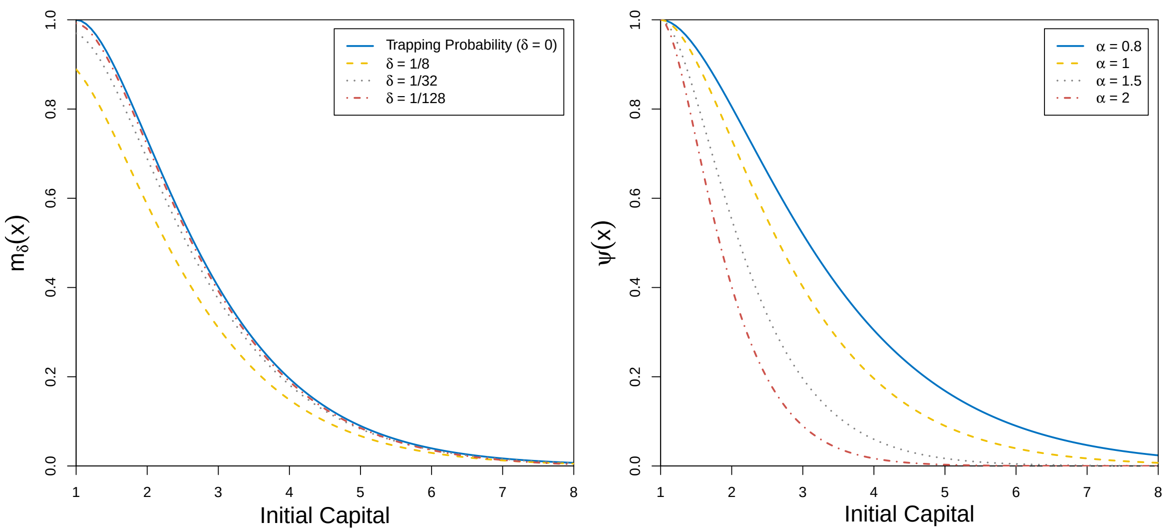
$$\psi(x) = \frac{1}{U\left(1-\frac{\lambda}{r},1-\frac{\lambda}{r};0\right)} e^{y(x)} U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda}{r}; -y(x)\right).$$

Figure 1(b) displays the trapping probability $\psi(x)$ for the stochastic capital process X_t. We can further simplify the expression for the trapping probability using the upper incomplete gamma function $\Gamma(a;z) = \int_z^\infty e^{-t}t^{a-1}dt$. Applying the relation

$$\Gamma(a;z) = e^{-z}U(1-a, 1-a; z)$$

(see equation (6.5.3) of Abramowitz and Stegun (1972)) and the fact that $\Gamma(a;0) = \Gamma(a)$ for $\mathbb{R}(a) > 0$, we have

$$\psi(x) = \frac{\Gamma\left(\frac{\lambda}{r}; -y(x)\right)}{\Gamma\left(\frac{\lambda}{r}\right)}. \tag{3.11}$$



(a) (b)

Figure 1: (a) Laplace transform $m_\delta(x)$ of the trapping time when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$ for $\delta = 0, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$ (b) Trapping probability $\psi(x)$ when $Z_i \sim Exp(\alpha)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$ for $\alpha = 0.8, 1, 1.5, 2$.

(ii) As an application of the Laplace transform of the trapping time, one particular quantity of interest is the expected trapping time. This can be obtained by taking the derivative of $m_\delta(x)$, where

$$\mathbb{E}\left[\tau_x\right] = -\left.\frac{d}{d\delta} m_\delta(x)\right|_{\delta=0}.$$

As such, we differentiate Tricomi's Confluent Hypergeometric Function with respect to its second parameter. Denote

$$U^{(c)}(a,c;z) \equiv \frac{d}{dc}U(a,c;z).$$

A closed form expression of the aforementioned derivative can be given in terms of series expansions, such that

$$\begin{aligned}
U^{(c)}(a,c;z) = {} & (\eta(a-c+1) - \pi\cot(c\pi))U(a,c;z) \\
& - \frac{\Gamma(c-1)z^{1-c}\log(z)}{\Gamma(a)} M(a-c+1, 2-c; z) \\
& - \frac{\Gamma(c-1)z^{1-c}}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{(a-c+1)_k(\eta(a-c+k+1) - \eta(2-c+k))z^k}{(2-c)_k k!} \\
& - \frac{\Gamma(1-c)}{\Gamma(a-c+1)} \sum_{k=0}^{\infty} \frac{\eta(c+k)(a)_k z^k}{(c)_k k!}, \qquad c \notin \mathbb{Z},
\end{aligned} \tag{3.12}$$

where $\eta(z) = \frac{d\ln[\Gamma(z)]}{dz} = \frac{\Gamma'(z)}{\Gamma(z)}$ corresponds to equation (6.3.1) of Abramowitz and Stegun (1972), also known as the digamma function. Thus, using expression (3.12), we obtain the expected trapping time

$$\begin{aligned}
\mathbb{E}[\tau_x] = {} & \frac{\Gamma\left(\frac{\lambda}{r}; -y(x)\right)}{\lambda U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda}{r}; 0\right)} - \frac{\Gamma\left(\frac{\lambda}{r}; -y(x)\right) U^{(c)}\left(1-\frac{\lambda}{r}, 1-\frac{\lambda}{r}; 0\right)}{r\left[U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda}{r}; 0\right)\right]^2} \\
& + e^{y(x)} \frac{U^{(c)}\left(1-\frac{\lambda}{r}, 1-\frac{\lambda}{r}; -y(x)\right)}{rU\left(1-\frac{\lambda}{r}, 1-\frac{\lambda}{r}; 0\right)}.
\end{aligned}$$

In line with intuition, the expected trapping time is an increasing function of both the capital growth rate r and initial capital x. However, since the capital process grows exponentially, large initial capital and capital growth rates significantly reduce the trapping probability and increase the expected trapping time to the point where it becomes non-finite, making the indicator function in the expected discounted penalty function (3.1) tend to zero. A number of expected trapping times for varying values of r are displayed in Figure 2.

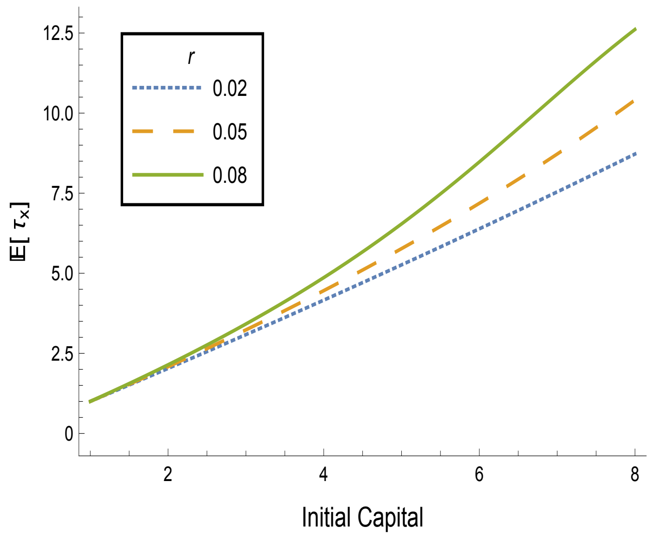


Figure 2: Expected trapping time when $Z_i \sim Exp(1)$, $\lambda = 1$ and $x^* = 1$ for $r = 0.02, 0.05, 0.08$.

(iii) The ruin probability for the insurance model (2.4) given by

$$\xi(u) = P(U_t < 0 \text{ for some } t > 0 \mid U_0 = u),$$

is found by Sundt and Teugels (1995) to satisfy the IDE

$$(au + p)\xi'(u) - \lambda\xi(u) + \lambda \int_0^u \xi(u - z)\, dG(z) + \lambda(1 - G(u)) = 0, \qquad u \geq 0. \tag{3.13}$$

Note that when $p = 0$, (3.13) coincides with the special case of (3.2) when $x^* = 0$, $w(x) = 1$, and $\delta = 0$. Thus, the household's trapping time can be thought of as the insurer's ruin time. Indeed, the ruin probability in the case of exponential claims when $p = 0$ as shown in Section 6 of Sundt and Teugels (1995) is exactly the same as the trapping probability (3.11) when $x^* = 0$.

4 Introducing Microinsurance

As in Kovacevic and Pflug (2011), we assume that households have the option of enrolling in a microinsurance scheme that covers a certain proportion of the capital losses they encounter. The microinsurance policy has proportionality factor $1-\kappa$, where $\kappa \in [0, 1]$, such that $100\cdot(1-\kappa)$ percent of the damage is covered by the microinsurance provider. The premium rate paid by households, calculated according to the expected value principle is given by

$$\pi(\kappa,\theta) = (1+\theta)\cdot(1-\kappa)\cdot\lambda\cdot\mathbb{E}(Z_i),$$

where θ is some loading factor. The expected value principle is popular due to its simplicity and transparency. When $\theta = 0$, one can consider $\pi(\kappa,\theta)$ to be the pure risk premium (Albrecher et al., 2017). We assume the basic model parameters are unchanged by the introduction of microinsurance coverage.

The stochastic capital process of a household covered by a microinsurance policy is denoted by $X_t^{(\kappa)}$. We differentiate between all variables and parameters relating to the original uninsured and insured processes by using the superscript (κ) in the latter case.

Since the premium is paid from a household's income, the capital growth rate r is adjusted such that it reflects the lower rate of income generation resulting from the need for premium payment. The premium rate is restricted to prevent certain poverty, which would occur should the premium rate exceed the rate of income generation. The capital growth rate of the insured household $r^{(\kappa)}$ is lower than that of the uninsured household, while the critical capital is higher.

In between jumps, where $T_{i-1} \leq t < T_i$, the insured stochastic growth process $X_t^{(\kappa)}$ behaves in the same manner as (2.2), with parameters corresponding to the proportional insurance case of this section, making particular note of the increased critical capital $x^{(\kappa)*}$:

$$X_t^{(\kappa)} = \begin{cases} \left(X_{T_{i-1}}^{(\kappa)} - x^{(\kappa)*}\right) e^{r^{(\kappa)}(t-T_{i-1})} + x^{(\kappa)*} & \text{if } X_{T_{i-1}}^{(\kappa)} > x^{(\kappa)*}, \\ X_{T_{i-1}}^{(\kappa)} & \text{otherwise.} \end{cases}$$

For $t = T_i$, the process is given by

$$X_{T_i}^{(\kappa)} = \begin{cases} \left(X_{T_{i-1}}^{(\kappa)} - x^{(\kappa)*}\right) e^{r^{(\kappa)}(T_i-T_{i-1})} + x^{(\kappa)*} - \kappa\cdot Z_i & \text{if } X_{T_{i-1}}^{(\kappa)} > x^{(\kappa)*}, \\ X_{T_{i-1}}^{(\kappa)} - \kappa\cdot Z_i & \text{otherwise.} \end{cases}$$

By enrolling in a microinsurance scheme, a household's capital losses are reduced to $Y_i := \kappa\cdot Z_i$. Considering the case in which losses follow an exponential distribution with parameter $\alpha > 0$, the structure of (3.3) remains the same. However, acquisition of a proportional microinsurance policy changes the parameter of the distribution of the random variable of the losses (Y_i). Namely, we have that $Y_i \sim Exp\left(\alpha^{(\kappa)}\right)$ for $\kappa \in (0,1]$, where $\alpha^{(\kappa)} := \frac{\alpha}{\kappa}$. We can therefore utilise the results obtained in Section 3 to obtain the Laplace transform of the trapping time for the insured process, which is given by

$$m_\delta^{(\kappa)}(x) = \frac{\lambda}{(\lambda+\delta)U\left(1-\frac{\lambda}{r^{(\kappa)}}, 1-\frac{\lambda+\delta}{r^{(\kappa)}}; 0\right)} e^{y^{(\kappa)}(x)} U\left(1-\frac{\lambda}{r^{(\kappa)}}, 1-\frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\kappa)}(x)\right), \tag{4.1}$$

where $y^{(\kappa)}(x) = -\alpha^{(\kappa)}\left(x - x^{(\kappa)*}\right)$. Figure 3(a) displays the Laplace transform $m_\delta^{(\kappa)}(x)$ for varying values of δ.

Remarks.

(i) The trapping probability of the insured process $\psi^{(\kappa)}(x)$, displayed in Figure 3(b), is given by

$$\psi^{(\kappa)}(x) = \frac{\Gamma\left(\frac{\lambda}{r^{(\kappa)}}; -y^{(\kappa)}(x)\right)}{\Gamma\left(\frac{\lambda}{r^{(\kappa)}}\right)}.$$

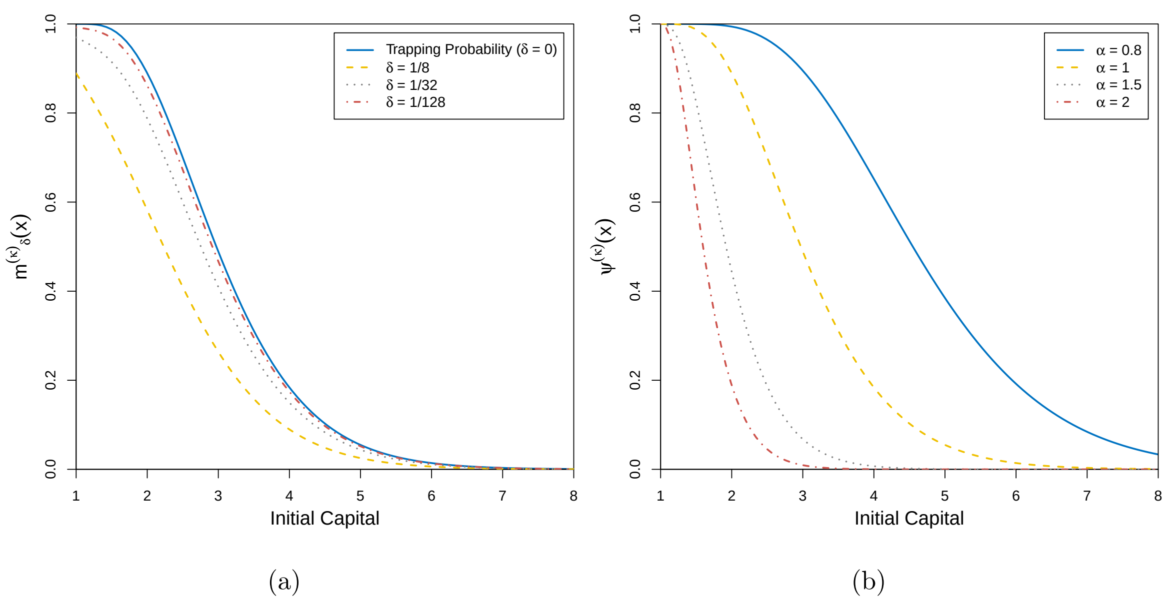


Figure 3: (a) Laplace transform $m_\delta^{(\kappa)}(x)$ of the trapping time when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^{(\kappa)*} = 1$, $\kappa = 0.5$ and $\theta = 0.5$ for $\delta = 0, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$ (b) Trapping probability $\psi^{(\kappa)}(x)$ when $Z_i \sim Exp(\alpha)$, $r = 0.5$, $\lambda = 1$, $x^{(\kappa)*} = 1$, $\kappa = 0.5$ and $\theta = 0.5$ for $\alpha = 0.8, 1, 1.5, 2$.

(ii) When $\kappa = 0$ the household has full microinsurance coverage, the microinsurance provider covers the total capital loss experienced by the household. On the other hand, when $\kappa = 1$, no coverage is provided by the insurer i.e., $X_t = X_t^{(\kappa)}$.

(iii) We are interested in studying significant capital losses, since low-income individuals are commonly exposed to this type of shock. Hence, throughout the paper, the parameter $\alpha > 0$ should be considered to reflect the desired loss behaviour.

Figure 4 presents a comparison between the trapping probabilities of the insured and uninsured processes. As in Kovacevic and Pflug (2011), households with initial capital close to the critical capital (here, the critical capital $x^* = 1$), i.e. the most vulnerable individuals, do not receive a real benefit from enrolling in a microinsurance scheme. Although subscribing to a proportional microinsurance scheme reduces capital losses, premium payments appear to make the most vulnerable households more prone to falling into the area of poverty. In Figure 4, the intersection point of the two probabilities corresponds to the boundary between households that benefit from the uptake of microinsurance and those who are adversely affected.

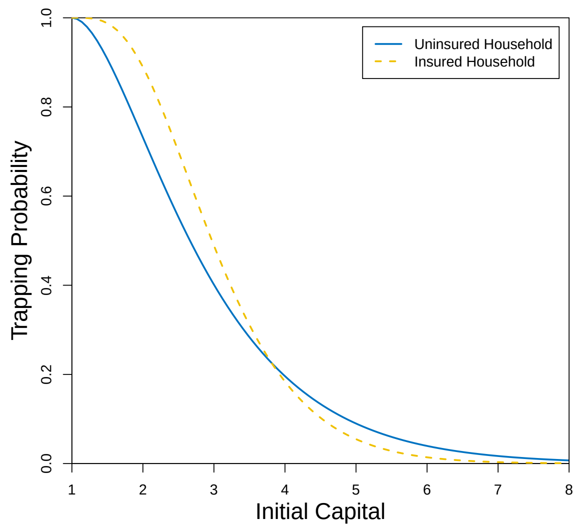


Figure 4: Trapping probabilities for the uninsured and insured capital processes, when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $\kappa = 0.5$, $\theta = 0.5$ and $x^* = 1$.

5 Microinsurance with Subsidised Constant Premiums

5.1 General Setting

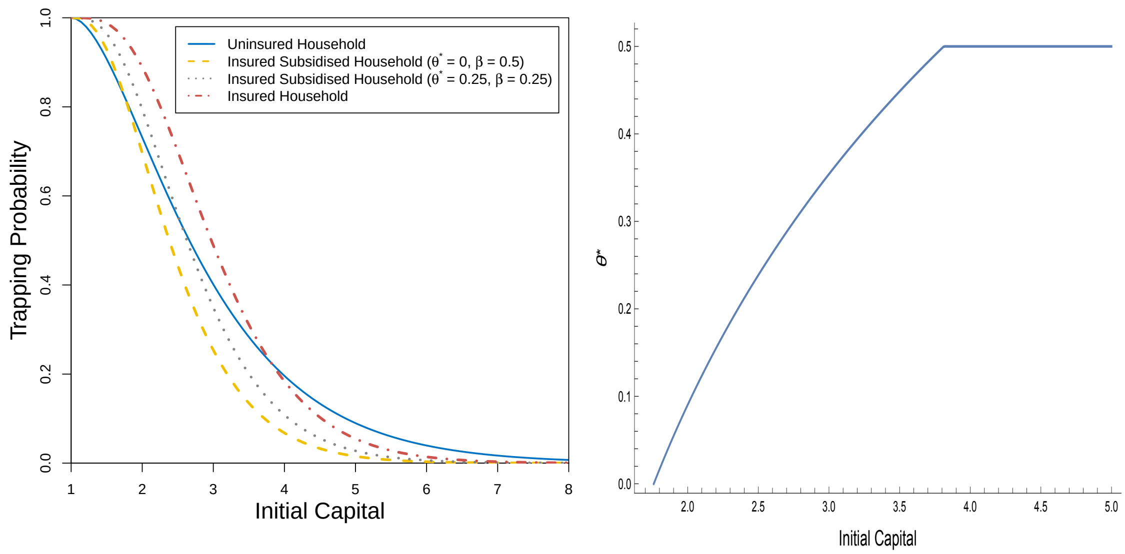
Since microinsurance alone is not enough to reduce the likelihood of impoverishment for those close to the poverty line, additional aid is required. In this section, we study the cost-effectiveness of government subsidised premiums, considering the case in which the government subsidises an amount $\beta = \theta - \theta^*$, while the microinsurance provider claims a lower loading factor θ^* (Kovacevic and Pflug, 2011). The following relationship between premiums for the non-subsidised and subsidised microinsurance schemes therefore holds

$$\pi(\kappa, \theta) = (1+\theta)\cdot(1-\kappa)\cdot\lambda\cdot\mathbb{E}(Z) \geq (1+\theta^*)\cdot(1-\kappa)\cdot\lambda\cdot\mathbb{E}(Z) = \pi(\kappa, \theta^*).$$

Naturally, we assume governments are interested in optimising the subsidy provided to households. Governments should provide subsidies to microinsurance providers such that they enhance households' benefits of enrolling in microinsurance schemes, however, they also need to gauge the cost-effectiveness of subsidy provision. Households with capital very close to the critical capital will not benefit from enrolling into the scheme even if the entire loading factor θ is subsidised by the government, however, more privileged households will. One approach to finding the optimal loading factor θ^* for households that could benefit from the government subsidy is to find the solution of the equation

$$\psi^{(\kappa,\theta^*)}(x) = \psi(x),$$

where $\psi^{(\kappa,\theta^*)}(x)$ and $\psi(x)$ denote the trapping probabilities of the insured subsidised and uninsured processes, respectively, since all loading factors below the optimal loading factor will induce a trapping probability lower than that of the uninsured process through a reduction in premium. This behaviour can be seen in Figure 5(a), while the "richest" households do not need help from the government since the non-subsidised insurance lowers their trapping probability below the uninsured case, the poorest individuals require more support. Moreover, as highlighted previously, there are households that do not receive any benefit from enrolling in the microinsurance scheme even when the government subsidises the entire loading factor (when households pay only the pure risk premium, this could occur if the government absorbs all premium administrative expenses). Note that Figure 5(b) illustrates the optimal loading factor θ^* for varying initial capital. Initial capitals are plotted from the point at which households begin benefiting from the subsidised microinsurance scheme, i.e. the point at which the dashed ($\theta = 0, \beta = 0.5$) line intersects the solid line in Figure 5(a). Additionally, Figure 5(b) verifies that, from the point at which the dashed-dotted (insured household) line intersects the solid line in Figure 5(a), the optimal loading factor remains constant, with $\theta^* = 0.5$, i.e. the "richest" households can afford to pay the entire premium.



(a) (b)

Figure 5: (a) Trapping probabilities for the uninsured, insured and insured subsidised capital processes when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$, $\kappa = 0.5$ and $\theta = 0.5$ for loading factors $\theta^* = 0, 0.25$ (b) Optimal loading factor θ^* for varying initial capitals when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$, $\kappa = 0.5$ and $\theta = 0.5$.

5.2 Cost of Social Protection

Next, we assess government cost-effectiveness for the provision of microinsurance premium subsidies to households. Let $\delta \geq 0$ be the force of interest for valuation, and let S denote the present value of all subsidies provided by the government until the trapping time such that

$$S = \beta \int_0^{\tau_x} e^{-\delta t} dt = \beta \bar{a}_{\overline{\tau_x}|}.$$

We assume a government provides subsidies according to the strategy introduced earlier, i.e. the government subsidises an amount $\beta = \theta - \theta^*$, while the microinsurance provider claims a lower loading factor θ^*.

For $x \geq x^{(\kappa,\theta^*)*}$, where $x^{(\kappa,\theta^*)*}$ denotes the critical capital of the insured subsidised process, let $V(x)$ be the expected discounted premium subsidies provided by the government to a household with initial capital x until trapping time, that is,

$$V(x) = \mathbb{E}\left[S \mid X_0^{(\kappa,\theta^*)*} = x\right].$$

Since $S = \frac{\beta}{\delta}\left[1 - e^{-\delta \tau_x}\right]$, we can define $m_\delta^{(\kappa,\theta^*)}(x)$, the Laplace transform of the trapping time with rate $r^{(\kappa,\theta^*)}$ and critical capital $x^{(\kappa,\theta^*)*}$, using the Laplace transform for the insured process previously obtained in (4.1) to compute $V(x)$ when losses are exponentially distributed with parameter $\alpha^{(\kappa)} > 0$. This yields

$$
\begin{aligned}
V(x) &= \frac{\beta}{\delta}\left[1 - m_\delta^{(\kappa,\theta^*)}(x)\right] \\
&= \frac{\beta}{\delta}\left[1 - \frac{\lambda}{(\lambda+\delta)U\left(1 - \frac{\lambda}{r^{(\kappa,\theta^*)}}, 1 - \frac{\lambda+\delta}{r^{(\kappa,\theta^*)}}, 0\right)} e^{y^{(\kappa,\theta^*)}(x)} U\left(1 - \frac{\lambda}{r^{(\kappa,\theta^*)}}, 1 - \frac{\lambda+\delta}{r^{(\kappa,\theta^*)}}, -y^{(\kappa,\theta^*)}(x)\right)\right],
\end{aligned}
$$

where $y^{(\kappa,\theta^*)}(x) = -\alpha^{(\kappa)}(x - x^{(\kappa,\theta^*)*})$. We now formally define the government's cost of social protection.

Definition 2. *Let $\psi^{(\kappa,\theta^*)}(x)$ be the trapping probability of a household enrolled in a subsidised microinsurance scheme with initial capital x. Additionally, let $M > 0$ be a constant representing the cost to lift households below the critical capital out of the area of poverty. The government's cost of social protection is given by*

$$
\textit{Cost of Social Protection} := V(x) + M \cdot \psi^{(\kappa,\theta^*)}(x).
$$

Remarks.

(i) For uninsured households, the government does not provide subsidies, i.e. $V(x) = 0$. Furthermore, we consider their trapping probability to be $\psi(x)$.

(ii) The government manages selection of an appropriate force of interest $\delta \geq 0$ and constant $M > 0$. For lower force of interest the government discounts future subsidies more heavily, while for higher interest future subsidies almost vanish. The constant M could be defined, for example, using the poverty gap index introduced by **?**, or in such a way that the government ensures with some probability that households will not fall into the area of poverty. Thus, higher values of M will increase the certainty that households will not return to poverty.

Figure 6 displays the government cost of social protection. Observe that in this particular example, we consider high values for both the force of interest δ and the constant M. The choice of M is motivated by Figure 4, which shows that from $x = 8$, the trapping probability for uninsured households is very close to zero. Note that a high value of δ hands a lower weight to future government subsidies whereas a high value of M grants higher certainty that a household will not return to the area of poverty once lifted out.

It is clear that governments do not benefit by entirely subsidising the "richest" households, since they will subsidise premiums indefinitely, almost surely (dashed line for highest values of initial capital). Hence, as illustrated in Figure 5(b), it is favourable for governments to remove subsidies for this particular group since their cost of social protection is even higher than when uninsured (solid line for highest values of initial capital). Conversely, governments perceive a lower cost of social protection when fully subsidising the loading factor θ for households with initial capital lying closer to the critical capital x^*. The cost of social protection when households pay only the pure risk premium is lower than when paying the premium entirely for values of initial capital in which the dashed line is below the dotted, in which case the government should support premium payments. However, due to the fact that they will almost surely fall into the area of poverty, requiring governments to pay the subsidy in addition to the cost of lifting a household out of poverty, it is not optimal to fully subsidise the loading factor for the most vulnerable, since the cost of social protection is higher than that for uninsured households. Note that, from the point of view of the governmental cost of social protection, Figure 6 confirms earlier statements asserting the inefficiency of providing premium support to the most vulnerable, i.e. neither individual households nor governments receive real benefit under such a scheme. Thus, alternative risk management strategies should be considered for this sector of the low-income population.

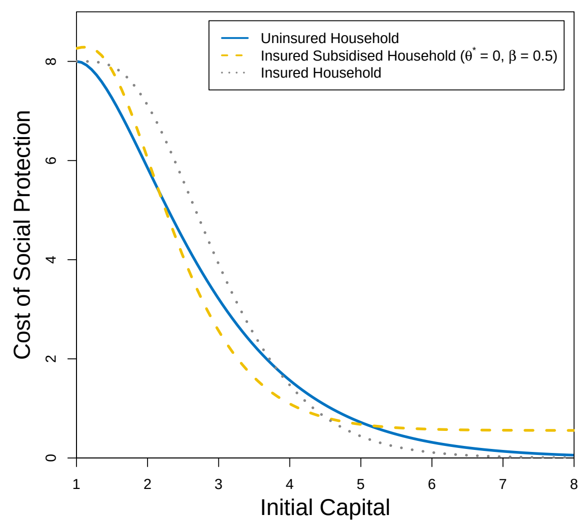


Figure 6: Cost of social protection for the uninsured, insured and insured subsidised capital processes when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$, $\kappa = 0.5$, $\theta = 0.5$, $\delta = 0.9$ and $M = 8$ for loading factor $\theta^* = 0$.

6 Microinsurance with Subsidised Flexible Premiums

6.1 General Setting

Since premiums are generally paid as soon as microinsurance coverage is purchased, a household's capital growth could be constrained. It is therefore interesting to consider alternative premium payment mechanisms. From the point of view of microinsurance providers, advance premium payments are preferred so that additional income can be generated through investment, naturally leading to lower premium rates. Conversely, consumers may find it difficult to pay premiums up front. This is a common problem in low-income populations, with consumers preferring to pay smaller installments over time (Churchill and Matul, 2006). Collecting premiums at a time that is inconvenient for households can be futile. Flexible premium payment mechanisms have been highly adopted by informal funeral insurers in South Africa, where policyholders pay premiums whenever they are able, rather than at a specific time during the month (Roth, 2000). Similar alternative insurance designs in which premium payments are delayed until the insured's income is realised and any indemnities are paid have also been studied. Under such designs, insurance take-up increases, since liquidity constraints are relaxed and concerns regarding insurer default, also prevalent in low-income classes, reduce (Liu, 2016).

In this section, we introduce an alternative microinsurance subsidy scheme with flexible premium payments. We denote the capital process of a household enrolled in the alternative microinsurance subsidy scheme by $X_t^{(\mathcal{A})}$. Furthermore, as in Section 4, we differentiate between variables and parameters relating to the original, insured and alternative insured processes using the superscript $(\mathcal{A})$. Under such an alternative microinsurance subsidy scheme, households pay premiums when their capital is above some capital barrier $B \geq x^{(\mathcal{A})*}$, with the premium otherwise paid by the government. In other words, whenever the insured capital process is below the capital level B, premiums are entirely subsidised by the government, however, when a household's capital is above B, the premium π is paid continuously by the household itself. This method of premium collection may motivate households to maintain a level of capital below B in order to avoid premium payments. Consequently, we assume that households always pursue capital growth. Our aim is to study how this alternative microinsurance subsidy scheme can help households reduce their probability of falling into the area of poverty. We also measure the cost-effectiveness of such scheme from the point of view of the government.

The intangibility of microinsurance makes it difficult to attract potential clients. Most clients will never experience a claim and so cannot perceive the real value of microinsurance, paying more to the scheme (in terms of premium payments) than what they actually receive from it. It is only when claims are settled that microinsurance becomes tangible. The alternative microinsurance subsidy scheme described here could increase client value, since, for example, individuals below the barrier B may submit claims, receive a payout and therefore perceive the value of microinsurance when they suffer a loss, regardless of whether they have ever paid a single premium. Other ways of increasing microinsurance client value include bundling microinsurance with other products and introducing Value Added Services (VAS), which represent services such as telephone hotlines for consultation with doctors or remote diagnosis services (for health schemes) offered to clients outside of the microinsurance contract (**?**).

Under the alternative microinsurance subsidy scheme, the Laplace transform of the trapping time satisfies the following differential equations:

$$0 = \begin{cases} -\frac{\left(x-x^{(\mathcal{A})*}\right)}{\alpha^{(\kappa)}} m_\delta^{(\mathcal{A})\prime\prime}(x) + \left[\frac{(\lambda+\delta-r)}{\alpha^{(\kappa)} r} - \left(x - x^{(\mathcal{A})*}\right)\right] m_\delta^{(\mathcal{A})\prime}(x) + \frac{\delta}{r} m_\delta^{(\mathcal{A})}(x) & \text{for } x^{(\mathcal{A})*} \leq x \leq B, \\ -\frac{\left(x-x^{(\mathcal{A})*}\right)}{\alpha^{(\kappa)}} m_\delta^{(\mathcal{A})\prime\prime}(x) + \left[\frac{(\lambda+\delta-r^{(\kappa)})}{\alpha^{(\kappa)} r^{(\kappa)}} - \left(x - x^{(\mathcal{A})*}\right)\right] m_\delta^{(\mathcal{A})\prime}(x) + \frac{\delta}{r^{(\kappa)}} m_\delta^{(\mathcal{A})}(x) & \text{for } x \geq B. \end{cases} \tag{6.1}$$

As in Section 3, use of the change of variable $y^{(\mathcal{A})} := y^{(\mathcal{A})}(x) = -\alpha^{(\kappa)}(x - x^{(\mathcal{A})*})$ leads to Kummer's Confluent Hypergeometric Equation and thus,

$$m_\delta^{(\mathcal{A})}(x) = \begin{cases} C_1 M\left(-\frac{\delta}{r}, 1 - \frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(x)\right) + C_2 e^{y^{(\mathcal{A})}(x)} U\left(1 - \frac{\lambda}{r}, 1 - \frac{\lambda+\delta}{r}; -y^{(\mathcal{A})}(x)\right) & \text{for } x^{(\mathcal{A})*} \leq x \leq B, \\ C_3 M\left(-\frac{\delta}{r^{(\kappa)}}, 1 - \frac{\lambda+\delta}{r^{(\kappa)}}; y^{(\mathcal{A})}(x)\right) + C_4 e^{y^{(\mathcal{A})}(x)} U\left(1 - \frac{\lambda}{r^{(\kappa)}}, 1 - \frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(x)\right) & \text{for } x \geq B, \end{cases}$$

for arbitrary constants $C_1, C_2, C_3, C_4 \in \mathbb{R}$. Under the boundary condition $\lim_{x\to\infty} m_\delta^{(\mathcal{A})}(x) = 0$ with asymptotic behaviour of the Kummer function $M(a, c; z)$ as presented in Section 3, we deduce that $C_3 = 0$. Also, since $m_\delta^{(\mathcal{A})}(x^{(\mathcal{A})*}) = \frac{\lambda}{\lambda+\delta}$, we obtain $C_1 = \frac{\lambda}{\lambda+\delta} - C_2 U\left(1 - \frac{\lambda}{r}, 1 - \frac{\lambda+\delta}{r}; 0\right)$.

Due to the continuity of the functions $m_\delta^{(\mathcal{A})}(x)$ and $m_\delta^{(\mathcal{A})\prime}(x)$ at $x = B$ and the differential properties of the Confluent Hypergeometric Functions

$$\frac{d}{dz} M(a, c; z) = \frac{a}{c} M(a+1, c+1; z),$$

$$\frac{d}{dz} U(a, c; z) = -aU(a+1, c+1; z),$$

upon simplification,

$$C_4 = \frac{\left[\frac{\lambda}{\lambda+\delta} - C_2 U\left(1 - \frac{\lambda}{r}, 1 - \frac{\lambda+\delta}{r}; 0\right)\right] M\left(-\frac{\delta}{r}, 1 - \frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right) + C_2 e^{y^{(\mathcal{A})}(B)} U\left(1 - \frac{\lambda}{r}, 1 - \frac{\lambda+\delta}{r}; -y^{(\mathcal{A})}(B)\right)}{e^{y^{(\mathcal{A})}(B)} U\left(1 - \frac{\lambda}{r^{(\kappa)}}, 1 - \frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(B)\right)}$$

and

$$C_2 = \frac{\frac{\lambda}{\lambda+\delta}\left[\frac{\delta\alpha^{(\kappa)}}{(r-\lambda-\delta)} M\left(1 - \frac{\delta}{r}, 2 - \frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right) + M\left(-\frac{\delta}{r}, 1 - \frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right)\left(\alpha^{(\kappa)} - D\right)\right]}{K},$$

where

$$D := \frac{\alpha^{(\kappa)}\left(\frac{\lambda}{r^{(\kappa)}} - 1\right) U\left(2 - \frac{\lambda}{r^{(\kappa)}}, 2 - \frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(B)\right)}{U\left(1 - \frac{\lambda}{r^{(\kappa)}}, 1 - \frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(B)\right)} \tag{6.2}$$

and

$$
\begin{aligned}
K :=\, & M\left(-\frac{\delta}{r}, 1-\frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right) U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; 0\right)\left(\alpha^{(\kappa)}-D\right) \\
& + De^{y^{(\mathcal{A})}(B)} U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; -y^{(\mathcal{A})}(B)\right) \\
& + \frac{\delta\alpha^{(\kappa)}}{(r-\lambda-\delta)} M\left(1-\frac{\delta}{r}, 2-\frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right) U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; 0\right) \\
& - \alpha^{(\kappa)} e^{y^{(\mathcal{A})}(B)}\left(\frac{\lambda}{r}-1\right) U\left(2-\frac{\lambda}{r}, 2-\frac{\lambda+\delta}{r}; -y^{(\mathcal{A})}(B)\right).
\end{aligned}
\tag{6.3}
$$

Remarks.

(i) The trapping probability $\psi^{(\mathcal{A})}(x)$ for the alternative microinsurance subsidy scheme is given by

$$
\psi^{(\mathcal{A})}(x) = \begin{cases} 1 - \dfrac{\Gamma\left(\frac{\lambda}{r}\right)-\Gamma\left(\frac{\lambda}{r};-y^{(\mathcal{A})}(x)\right)}{\left(-y^{(\mathcal{A})}(B)\right)^{\lambda\left(\frac{1}{r}-\frac{1}{r^{(\kappa)}}\right)}\Gamma\left(\frac{\lambda}{r^{(\kappa)}};-y^{(\mathcal{A})}(B)\right)+\Gamma\left(\frac{\lambda}{r}\right)-\Gamma\left(\frac{\lambda}{r};-y^{(\mathcal{A})}(B)\right)}, & x \leq B \\[2ex] \dfrac{\left(-y^{(\mathcal{A})}(B)\right)^{\lambda\left(\frac{1}{r}-\frac{1}{r^{(\kappa)}}\right)}\Gamma\left(\frac{\lambda}{r^{(\kappa)}};-y^{(\mathcal{A})}(x)\right)}{\left(-y^{(\mathcal{A})}(B)\right)^{\lambda\left(\frac{1}{r}-\frac{1}{r^{(\kappa)}}\right)}\Gamma\left(\frac{\lambda}{r^{(\kappa)}};-y^{(\mathcal{A})}(B)\right)+\Gamma\left(\frac{\lambda}{r}\right)-\Gamma\left(\frac{\lambda}{r};-y^{(\mathcal{A})}(B)\right)}, & x \geq B. \end{cases}
$$

Similar to the subsidised case, we can find the optimal barrier B^* by determining the solution of the equation

$$
\psi^{(\mathcal{A},B^*)}(x) = \psi(x),
$$

where $\psi^{(\mathcal{A},B^*)}(x)$ and $\psi(x)$ denote the trapping probability of the capital process under the alternative microinsurance subsidy scheme and the uninsured capital process, respectively. Some examples are presented after the remarks.

(ii) When $B \to x^{(\mathcal{A})*}$, the trapping probability for the alternative microinsurance subsidy scheme is equal to the trapping probability obtained for the insured case $\psi^{(\kappa)}(x)$, i.e.

$$
\lim_{B\to x^{(\mathcal{A})*}} \psi^{(\mathcal{A})}(x) = \frac{\Gamma\left(\frac{\lambda}{r^{(\kappa)}}; -y^{(\kappa)}(x)\right)}{\Gamma\left(\frac{\lambda}{r^{(\kappa)}}\right)}.
$$

Moreover, letting $B \to \infty$, the trapping probability is given by

$$
\lim_{B\to\infty} \psi^{(\mathcal{A})}(x) = \frac{\Gamma\left(\frac{\lambda}{r}; -y^{(\kappa)}(x)\right)}{\Gamma\left(\frac{\lambda}{r}\right)}.
$$

(iii) Figure 7 displays the expected trapping time under the alternative microinsurance subsidy scheme. Not surprisingly, the expected trapping time is an increasing function of both the capital growth rate r and barrier B.

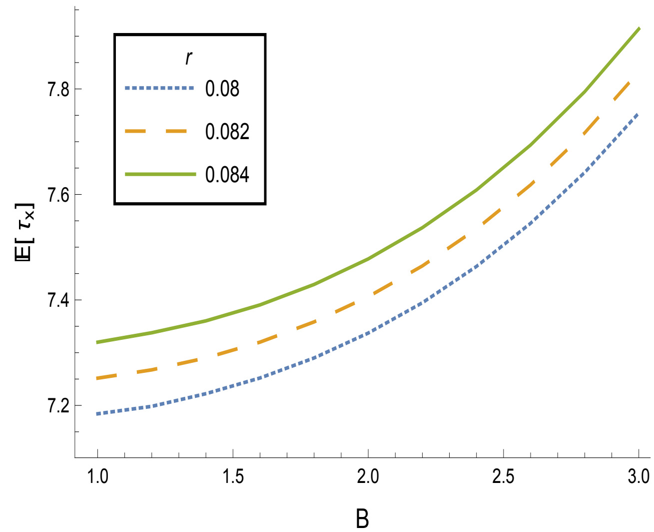
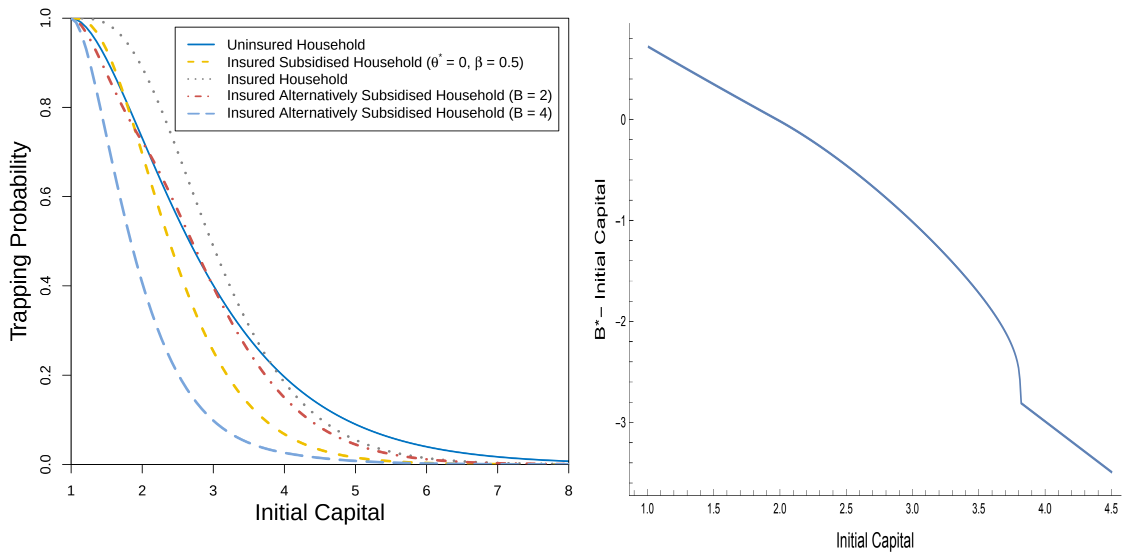


Figure 7: Expected trapping time when $Z_i \sim Exp(1)$, $\lambda = 1$, $x = 3.5$, $x^{(\mathcal{A})*} = 1$, $\kappa = 0.5$ and $\theta = 0.5$ for $r = 0.08, 0.082, 0.084$.

Figure 8(a) shows the trapping probabilities for varying initial capital values under the uninsured, insured, subsidised and alternatively subsidised schemes. As expected, increasing the value of the capital barrier B helps households to reduce their probability of falling into the area of poverty, since support from the government is received when their capital resides in the region between the critical capital $x^{(\mathcal{A})*}$ and the barrier B. Furthermore, as in the previous case, households with higher levels of initial capital do not need support from the government, insurance without subsidies decreases their trapping probability to a level below the uninsured (households with initial capital greater than or equal to the point at which the dotted line intersects the solid line).

The optimal barrier for these individuals is in fact the critical capital, i.e., $B^* = x^{(\mathcal{A})*}$, households with higher initial capital can therefore afford to pay for microinsurance coverage themselves, as illustrated in Figure 8(b). Figure 8(b) also shows that for the most vulnerable, the government should set up a barrier above their initial capital to remove capital growth constraints associated with premium payments. This level should be selected until the household reaches a capital level that is adequate in ensuring their trapping probability will be equal to that of an uninsured household. Conversely, for the more privileged (those in Figure 8(b), with initial capital approximately greater than or equal to 2), the government should establish barriers below their initial capital, with households paying premiums themselves as soon as they enrol in the microinsurance scheme. This behaviour is mainly due to the fact that their level of capital is distant from the critical capital $x^{(\mathcal{A})*}$. These households are unlikely to fall into the area of poverty after suffering one capital loss, they are instead likely to fall into the region between the critical capital and the barrier B (i.e. the area within which the government pays premiums), before entering the area of poverty. Thus, the aforementioned region acts as a "buffer" for households, since once in this region they will benefit from coverage without paying any premiums. Increasing the initial capital will lead to a decrease in the size of the "buffer" region until it disappears when $B = x^{(\mathcal{A})*}$, as shown in the lower right corner of Figure 8(b), where a straight line is visible.



(a) (b)

Figure 8: (a) Trapping probabilities for the uninsured, insured, insured subsidised with $\theta^* = 0$ and insured alternatively subsidised with $B = 2, 4$ capital processes when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$, $\kappa = 0.5$ and $\theta = 0.5$ (b) Difference between the optimal barrier and the initial capital, i.e. $B^* - x$, for varying initial capitals, when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^{(\mathcal{A})*} = 1$, $\kappa = 0.5$ and $\theta = 0.5$.

6.2 Cost of Social Protection

Similarly to the previous section, it is reasonable to measure the governmental cost-effectiveness of providing microinsurance premium subsidies to households under the alternative microinsurance subsidy scheme. For this reason, we define $V^{(\mathcal{A})}(x)$ as the expectation of the present value of all subsidies provided by the government until the trapping time under the alternative microinsurance subsidy scheme, that is

$$V^{(\mathcal{A})}(x) := \mathbb{E}\left[\int_0^{\tau_x} \pi e^{-\delta t} \mathbb{1}_{\left\{X_t^{(\mathcal{A})} < B\right\}} dt \middle| X_0^{(\mathcal{A})} = x\right].$$

If the derivative exists, then using standard infinitesimal generator arguments for $X_t^{(\mathcal{A})}$, one gets the following IDE for $V^{(\mathcal{A})}(x)$ under the barrier B

$$r(x-x^{(\mathcal{A})*})V^{(\mathcal{A})\prime}(x)-(\lambda+\delta)V^{(\mathcal{A})}(x)+\lambda\int_0^{x-x^{(\mathcal{A})*}}V^{(\mathcal{A})}(x-z)dG(z)+\pi=0, \qquad x^{(\mathcal{A})*}\le x\le B. \tag{6.4}$$

Hence, assuming $Z_i\sim Exp(\alpha^{(\kappa)})$, the function satisfies the nonhomogeneous differential equation given by

$$-\frac{(x-x^{(\mathcal{A})*})}{\alpha^{(\kappa)}}V^{(\mathcal{A})\prime\prime}(x)+\left[\frac{(\lambda+\delta-r)}{\alpha^{(\kappa)}r}-(x-x^{(\mathcal{A})*})\right]V^{(\mathcal{A})\prime}(x)+\frac{\delta}{r}V^{(\mathcal{A})}(x)-\frac{\pi}{r}=0, \qquad x^{(\mathcal{A})*}\le x\le B. \tag{6.5}$$

Letting $V_h^{(\mathcal{A})}(x)$ be the homogeneous solution of (6.5), we have

$$V_h^{(\mathcal{A})}(x)=R_1M\left(-\frac{\delta}{r},1-\frac{\lambda+\delta}{r};y^{(\mathcal{A})}(x)\right)+R_2e^{y^{(\mathcal{A})}(x)}U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};-y^{(\mathcal{A})}(x)\right), \quad x^{(\mathcal{A})*}\le x\le B,$$

for arbitrary constants $R_1,R_2\in\mathbb{R}$, where $y^{(\mathcal{A})}(x)=-\alpha^{(\kappa)}(x-x^{(\mathcal{A})*})$.

Since the general solution of (6.5) can be written as

$$V^{(\mathcal{A})}(x)=V_h^{(\mathcal{A})}(x)+V_p^{(\mathcal{A})}(x),$$

where $V_p^{(\mathcal{A})}(x)$ is a particular solution, one can easily verify that $V_p^{(\mathcal{A})}(x)=\frac{\pi}{\delta}$ for all $x^{(\mathcal{A})*}\le x\le B$. Then, letting $x=x^{(\mathcal{A})*}$ in (6.4) yields $V^{(\mathcal{A})}(x^{(\mathcal{A})*})=\frac{\pi}{\lambda+\delta}$ and subsequently

$$R_1=-\left[\frac{\lambda\pi}{(\lambda+\delta)\delta}+R_2U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};0\right)\right].$$

For $x^{(\mathcal{A})*}\le x\le B$, we therefore have

$$\begin{aligned}V^{(\mathcal{A})}(x)=&-\left[\frac{\lambda\pi}{(\lambda+\delta)\delta}+R_2U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};0\right)\right]M\left(-\frac{\delta}{r},1-\frac{\lambda+\delta}{r};y^{(\mathcal{A})}(x)\right)\\&+R_2e^{y^{(\mathcal{A})}(x)}U\left(1-\frac{\lambda}{r},1-\frac{\lambda+\delta}{r};-y^{(\mathcal{A})}(x)\right)+\frac{\pi}{\delta}.\end{aligned}$$

Above the barrier B, $V^{(\mathcal{A})}(x)$ satisfies (6.1) for $x \geq B$, and so

$$V^{(\mathcal{A})}(x) = R_3 M\left(-\frac{\delta}{r^{(\kappa)}}, 1-\frac{\lambda+\delta}{r^{(\kappa)}}; y^{(\mathcal{A})}(x)\right) + R_4 e^{y^{(\mathcal{A})}(x)} U\left(1-\frac{\lambda}{r^{(\kappa)}}, 1-\frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(x)\right), \qquad x \geq B,$$

for arbitrary constants $R_3, R_4 \in \mathbb{R}$. Since $\lim_{x\to\infty} V^{(\mathcal{A})}(x) = 0$ by definition, we have that $R_3 = 0$. Using the continuity of the functions $V^{(\mathcal{A})}(x)$ and $V^{(\mathcal{A})\prime}(x)$ at $x = B$ and the differential properties of the Confluent Hypergeometric Functions,

$$\begin{aligned} R_4 = & \frac{-\left[\frac{\lambda\pi}{(\lambda+\delta)\delta} + R_2 U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; 0\right)\right] M\left(-\frac{\delta}{r}, 1-\frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right)}{e^{y^{(\mathcal{A})}(B)} U\left(1-\frac{\lambda}{r^{(\kappa)}}, 1-\frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(B)\right)} \\ & + \frac{R_2 e^{y^{(\mathcal{A})}(B)} U\left(1-\frac{\lambda}{r}, 1-\frac{\lambda+\delta}{r}; -y^{(\mathcal{A})}(B)\right) + \frac{\pi}{\delta}}{e^{y^{(\mathcal{A})}(B)} U\left(1-\frac{\lambda}{r^{(\kappa)}}, 1-\frac{\lambda+\delta}{r^{(\kappa)}}; -y^{(\mathcal{A})}(B)\right)} \end{aligned}$$

and

$$R_2 = -\frac{\frac{\lambda\pi}{\lambda+\delta}\left[\frac{\alpha^{(\kappa)}}{(r-\lambda-\delta)} M\left(1-\frac{\delta}{r}, 2-\frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right) + \frac{1}{\delta} M\left(-\frac{\delta}{r}, 1-\frac{\lambda+\delta}{r}; y^{(\mathcal{A})}(B)\right)\left(\alpha^{(\kappa)} - D\right)\right] + \frac{\pi}{\delta}\left(D - \alpha^{(\kappa)}\right)}{K},$$

where D and K are (6.2) and (6.3), respectively.

Figure 9 compares the cost of social protection for the uninsured, insured, insured subsidised and insured alternatively subsidised households. Cost of social protection for the most vulnerable is not reduced with microinsurance coverage (dotted, dashed and dash-dotted lines are all above the solid line for initial capitals close to the critical capital x^*). As mentioned previously, this corresponds to the high trapping probability of this section of the population, with governments almost surely needing to lift these households out of the area of poverty through payment of a certain amount M, in addition to paying subsidies. On the other hand, the cost of social protection could be reduced by providing subsidies to households with greater levels of initial capital (when dashed and dash-dotted lines are below the solid line). Greater initial capitals lead to lower trapping probabilities and a reduction in the likelihood of the government need to pay the value M. As a result, the cost of social protection decreases even though subsidies are provided.

Also illustrated in Figure 9 is the greater cost-effectiveness of conditionally subsidising premiums, in comparison to proportional subsidisation (dashed-dotted line below all other lines for the majority of households in this particular group). Besides lowering the cost of social protection compared to that of uninsured households, it outperforms the traditional insured subsidised case in which some of the loading factor is absorbed by the government. Moreover, the alternative scheme eliminates the disadvantage of paying subsidies indefinitely for the "richest" households almost surely. In consequence, implementing a microinsurance scheme with barrier strategy reduces both the probability of households falling into the area of poverty and the governmental cost of social protection (shown in Figure 8(a) and Figure 9, respectively).

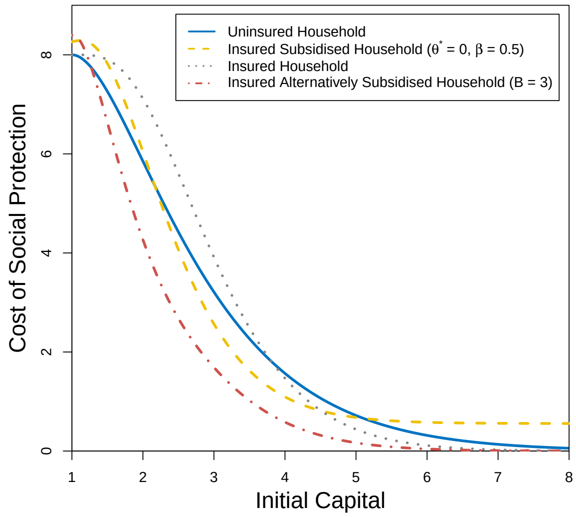


Figure 9: Cost of social protection for the uninsured, insured, insured subsidised with $\theta^* = 0$ and insured alternatively subsidised with $B = 3$ capital processes, when $Z_i \sim Exp(1)$, $r = 0.5$, $\lambda = 1$, $x^* = 1$, $\kappa = 0.5$, $\theta = 0.5$, $\delta = 0.9$ and $M = 8$.

7 Conclusion

Comparing the impact of three microinsurance mechanisms on the trapping probability of low-income households, we provide evidence for the importance of governmentally supported inclusive insurance in the strive towards poverty alleviation. The results of Sections 4 and 5 support those of Kovacevic and Pflug (2011), highlighting a threshold below which insurance increases the probability of trapping. Further to these findings, we have introduced an alternative mechanism with the capacity to reduce this effect, while strengthening government social protection programs by lowering costs.

Analysis of the subsidised microinsurance scheme with premium payment barrier, suggests that in general, the trapping probability of a household is reduced in comparison to basic microinurance and subsidised microinsurance structures, in addition to that of uninsured households. More significant influence is observed in relation to the governmental cost of social protection, with the cease of subsidy payments when household capital is sufficient facilitating government savings and therefore increasing social protection efficiency. Cost of social protection for those closest to the area of poverty remains greater than the corresponding uninsured cost in both subsidised schemes considered. For such households, governments must account for both their support of premium payments and the likely need for household removal from poverty. Government endorsement of further alternative risk mitigation strategies, such as asset accumulation, would be beneficial for those with capital close to the critical level, minimising their risk of falling beneath the poverty line, while reducing social protection costs.

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